TELECOMMUNICATIONS NETWORK MODERNIZATION. A LINEAR PROGRAMMING FORMULATION.

by

STAVROS CH. KOUZELIS, B.Sc. Athens University, Greece.

DEPARTMENT OF ELECTRICAL ENGINEERING.

McGill University

Montreal, Canada

May, 1981

ABSTRACT

A problem encountered in telephone network planning is the choice of timing and equipment types to be used to undertake the modernization of the network. There are many options: 1. Retain network in its existing analog state, 2. Introduce digital equipment in the Switching Center, 3. Enhance analog lines with some kind of modern equipment, 4. Replace present network with an advanced digital network, 5. Combinations of the above.

In this project an aggregate model of a modernization process is presented. The economic evaluator used in this cost analysis is the Net present Value (NPV) of cash flows which is optimized subject to capital budget constraints imposed on the modernized lines each year. Finally a linear program is used to solve for the optimal evolution strategy based on some numerical values of the input parameters.

The motivation for developing this model arose from the interest in relating modernization policies for a "Wire-Center" of a local telephone network to an appropriate economic indicator and to determine the optimal policy subject to a capital budget constraint.

TABLE OF CONTENTS

ACKNOWLEDGMENTS.

I wish to express my sincere thanks to Dr. Mason, my supervisor, who suggested the problem to work on and was a source of guidance throughout this work. Special thanks are also due to my advisor, Dr. Kabal, for his sustained help and encouragement.

Finally, I wish to express my deepest gratitude to my fiancee, Lena, for her sincere love and my parents who have guided me and supported me through the years.

LIST OF FIGURES.

LIST OF TABLES.

SECTION 1. PROLOGUE.

1.1 INTRODUCTION

In this paper, evolution policies for a telecommunications network modernization problem are presented. The model analyzed provides the cost criterion corresponding to a modernization policy, which may consist of the following or some combinations of them.

a. Retain network in its existing analog state.

b. Introduce digital equipment in the Switching Center.

c. Enhance analog lines with some kind of modern equipment.

d. Replace present network with an advanced digital network.

Some plausible assumptions have been made in order to keep the network model in a simple and understandable form.

From the stand point of costs, two types of switches are considered; the analog space switch and the digital time switch. Several types of interfaces have been assumed in this network model, depending upon the network state.

There are three service categories. First the conventional telephone services in an analog environment with a possible digital switch in the central office. Second the enhanced services, referred to also as narrow band services + POTS which represents a step beyond conventional telephony, carrying voice and data signals. Third, the most advanced services are the replaced or wide band + POTS services, offering telephone services as well as high speed digital communication services.

The terminals employed depend on the evolution state of the network. The system may include telephone sets, low speed data terminals, or high speed terminals.

From the economic analysis presented next, it will be shown that the evaluator used, the Net Present Value (NPV) of cash flows, has a complex formula and an analytical approach to the problem seems to be difficult.

However, some results have been obtained algorithmically, by using a computer to solve for the optimum policy corresponding to a given set of parameters. The results are input dependent but all data sets used indicate that pure modernization policies are preferable. This is a numerical result and not a rigorous proof. To generalize this observation more research is necessary and remains an open question for further investigation. Pure policy is the policy which is time invariant. This is a global optimum in the absence budget constraints, when capital budget constraints are imposed on the network evolution the optimal constrained solution is selected. A summary of these results is given in section 5.

SECTION 2. MODEL DESCRIPTION.

AN AGGREGATE ECONOMIC MODEL FOR INTEGRATED WIRE-CENTER MODERNIZATION.

An economic model will be derived for the evolution of a telecommunications network where the universe of Subscriber lines is divided into five distinct categories to be described below.

Three types of services are offered with these lines.

1. POTS. This is the Plain Ordinary telephone services.
2. Narrow Band Data + POTS services or enhanced services.
3. Wide Band Data + POTS services or replaced services.

To tackle this integrated Wire Center problem and keep it at an acceptable level of complexity, some assumptions and simplifications have been made.

Two generic types of local switches are considered; namely, analog and digital (fig .1). This is an idealization in that there are various types of analog and digital switches. However, it is assumed that the selection of switch types has been optimized within each class. Also no distinction is made between lines served by a digital line concentrator and those served directly from the digital switch. Again it is assumed that these choices have been optimized and the cost figures used reflect this optimal choice.

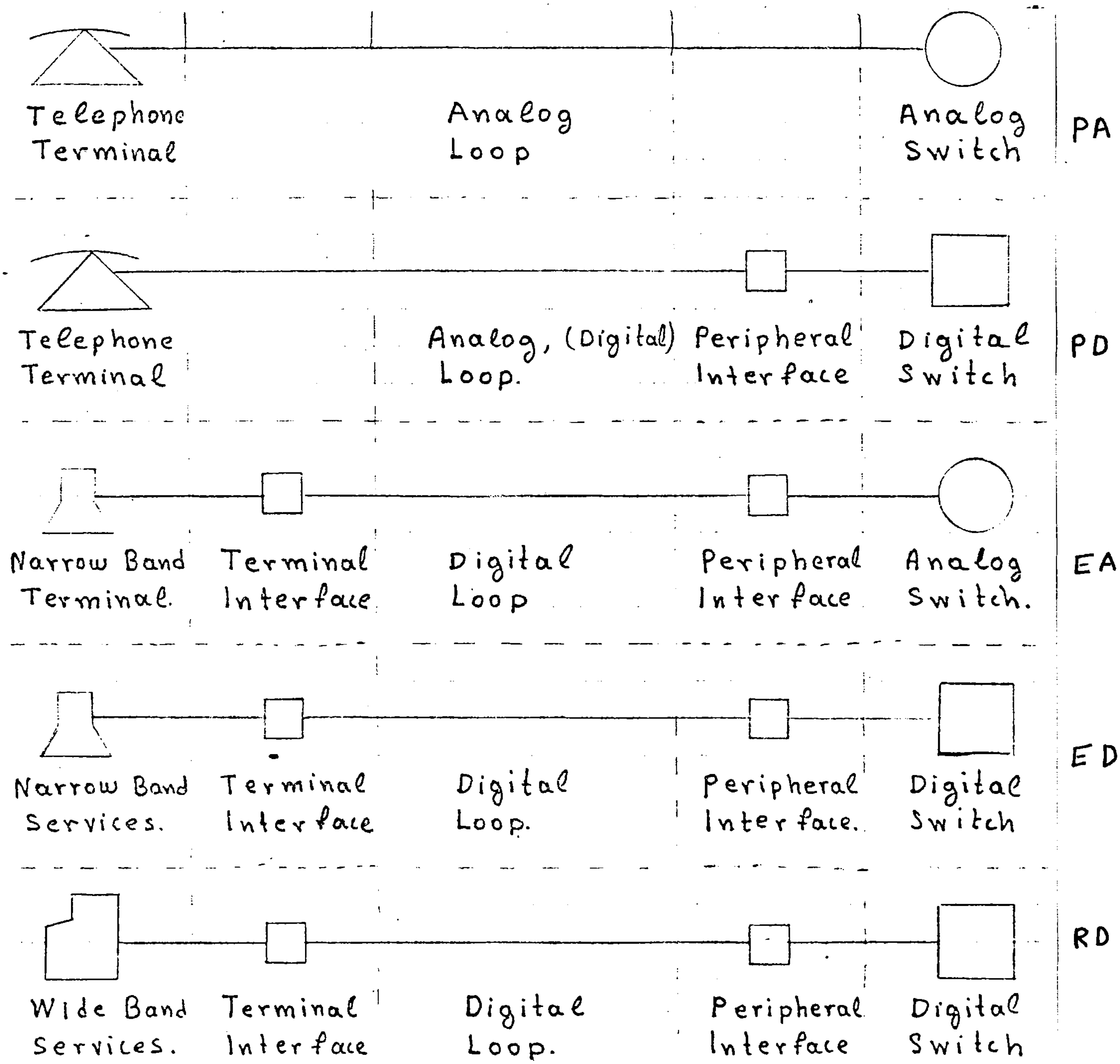

Fig .1 The five states of network evolution.

The above simplification has been made to investigate some global effects of the state dynamics, which include the effect of integrated introduction of loop services and switch facilities by Wire Center on network economics. An accurate model taking into account a more detailed switch and loop characterization would necessitate the inclusion of spatial as well as multilocation aspects.

Additional equipment must be installed at the terminal and the switch ends to provide the enhanced or the Wide Band services. A new loop is also required to provide the Wide Band services. (i.e. the existing loop must be replaced by fiber or coax).

As indicated above POTS and narrow band+ POTS services can be provided either by a digital switch or by an analog switch. The revenues associated with this service category will be the same in each case. However the costs will differ, as the electronics required will not likely be the same in each case.

The model considers only a digital switching option for the wide band services. This model is initiated at a 100 per cent Analog POTS, but it is possible with minor changes to initiate the network in some other state.

An overlay option analysis can be based on this model assuming that there is no coupling among lines in the two partitions. No loss of generality results from such an assumption since the population can be partitioned into two subpopulations namely:

1. Initial lines whose number remains constant throughout study horizon; and they are evolved in this study period.

2. Growth lines which increase linearly with time.

A free end point dynamics, discrete time formulation of the policy optimization problem based on the cash flows per year is considered. For this study a finite time horizon is selected, and it is generally taken to be sufficiently long to include the significant contributions to the evaluator. Start-up costs for the switch are not explicitly added but are included as a component of the capital cost per line for switching.

The expenditures on capital in a given year are constrained by a corresponding capital budget availability specified as an inequality.

Coupling among subpopulations

If the universe of lines is subdivided into 1. Existing lines served by analog switches providing POTS services and 2. Growth lines, some error can result if the populations are considered independently due to the ommision of certain couplings.

Coupling between decisions in these two subpopulations takes place via the start-up charges and possible service coverage constraints. The latter coupling may require equivalent service offerings for all lines (both existing and growth) in the given wire center.

Coupling among subpopulations can also arise if equipment reuse is permitted. For example if existing analog POTS lines are replaced by wide band digital lines, portions of the analog POTS lines such as terminal equipment might have some salvage value, as they can be reused elsewhere in the network for analog POTS.

Concluding this general description, it is obvious, that a realistic model taking into account the above modes of coupling will be complex. An upper bound on the network Net Present Value can be obtained by optimizing each subuniverse independently. Lower bounds can be obtained by applying the optimal decisions obtained for each subpopulation when considered in isolation to the whole population.

2.2 DEFINITIONS AND NOTATIONS

A line is defined as the equipment ensemble necessary for one call set up. A line is partitioned into 1.Subscribers terminal, 2.Outside plant loop, and 3.Corresponding switching equipment at the Central Office.

The line evolution is defined as the series of possible transitions a line undergoes from its initial Pots Analog state, through Pots digital, Enhanced Analog, Enhanced Digital, to reach finally the Replaced Digital state, given that everything occured within the study horizon.

However, there are cases where the optimum solution is obtained by leaving the lines in their original analog state without any modernization. In other words there has not been imposed the constraint that all lines have to reach the final state by the end of study period. This is referred to as a free end point problem. These types of lines do not quite follow the previously defined line evolution. Even though these lines are assumed to reach the final state <u>one</u> year after the study period, specifically at year h. This kind of evolution is called line Semi-evolution. Therefore the problem remains free end point inside the study period but is evolved to RD in the next year and it is no longer free end point.

Some important parameters used in this analysis are defined next.

h-1	End of study period.
0,1,2...h-1	Study years starting time 0
i	Discrete discount rate (*)
w	=1+i function of discount rate
f	=1/(1-w**(-1)) function of w
d	Digitization time
e	Enhancement time
r	Replacement time
d e r	Are the line evolution variables
N(e,d,r)	Number of lines enhanced at time e digitazed at time d replaced at time r
N(e,d,r)	Are called also renewal variables
T(k)	Capital for year k allocated by the firm to network modernization

* When a uniform rate of inflation is considered with an exponential growth rate j, an equivalent discount rate i' can be used where

$$\frac{1}{1+i'} = \frac{1+j}{1+i}$$

<u>States of the network</u>

PA	Pots Analog state
PD	Pots digital state

EA	Enhanced Analog state
ED	Enhanced Digital state
RD	Replaced Digital state

Cash flow parameters.

Due to the vast number of parameters employed, a notational convention is used to represent the parameters required. To obtain these parameters, prefixes can be attached to above "state names" to indicate costs

E	Indicates marginal (per line per year) Expense cost of some line type.
C	Indicates marginal (per line) Capital cost of some type.
S	Indicates marginal (per line) Salvage cost of some type.
R	Indicates marginal (per line per year) Revenues cost of some type.

Line partitioning

To discriminate among the various cost elements of a line in each state, suffixes have to be attached to previously defined "state names" to specify the portion of the line for which the given cost is intented.

_T	Specifies the _Terminal portion of line
-L	Specifies the _Loop portion of line
_S	Specifies the _switch portion of line

As an example EPD_T represents the Expense costs for a POTS Digital Terminal, or CED is the total capital for an Enhanced digital Terminal + Loop + Switch.

2.3 NETWORK STATE TRANSITIONS.

According to the assumptions made there are five possible states, namely PA, PD, EA, ED, RD.

The number of feasible state transitions are 9 as is depicted in the Fig .2

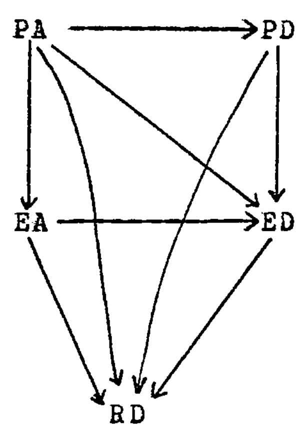

Fig .2 Network state transitions.

The kind of equipment introduced in the network depends on the state transition considered. An analytic description of all possible transitions is summarized in Fig . 1. Typical evolution policies include more than one state transitions as is shown next.

a. PA—>PD

Replace the local analog switch by a digital switch. A remote concentrator may be included to relieve feeder cables, if this results to some savings from the outside plant. Salvage analog switch.

b. PA—>EA

Place interfaces at the terminal and the Central Office. Line conditioning may also be necessary.

c. PA-->ED

Place a digital switch in the local office. Place a new terminal or electronics on the old one to interface with the digital switch. Salvage the analog switch. The introduced electronics may differ in cost with ones in PA-->EA transition above.

d. PA-->RD

Replace analog switch, terminal, and loop, by new terminal, fiber loop, and digital switch.

e. PD-->RD

Place new terminal and fiber loop.

Salvage terminal and and switch electronics, no savings from loop

f. PD-->ED

Place electronics at the terminal and at the switch associated with the Narrow Band services.

g. ED-->RD

Salvage terminal, loop, and electronics. Place new terminal, fibre loop, and switching equipment for Wide Band services.

h. EA-->ED

Salvage analog switch, and analog electronics. Place new switch, and digital Narrow Band electronics

i. EA-->RD

Salvage terminal, analog Narrow Band electronics. Place new terminal, fibre loop, and digital switch, and Central Office equipment for Wide band services.

2.4 CONSTRAINTS FOR LINE EVOLUTION VARIABLES

The line evolution or strategic variables e, d, r obey some constraints concerning their time ordering. All variables are taken to be non negative integers which are restrained not to exceed the value of study period h-1 plus 1 i.e. h to comprise all possible transitions.

The replacement state is the final state of the network evolution. Therefore the replacement time must come not prior to either e or d. In this model the POTS and Enhanced service categories can be supported by an analog or digital switch. This leaves unconstrained the time ordering of digitization and enhancement. The inequalities indicated previously are summarized in the following relation

$$0 \leq d, e \leq r \leq h$$

The d, e time ordering specifies two distinct classes of network evolution considered here. If e occurs prior to d the following physical events occur:

At e time

PA-->EA

Place analog electronics at the terminal and the Central Office with possible line conditioning.

At d time

EA-->ED

Salvage analog switch and analog electronics. Place digital switch and digital electronnics.

If on the other hand d preceds e the following physical events occur.

At d time

PA-->PD

Salvage analog switch. Place digital switch.

For e time

PD-->ED

Place digital electronics

According to above notation for the line evolution variables, we can identify the evolution paths with the following conditions on the variables.

PA-->PA	<==>	$0 \le e = d = r = h$
PA-->RD	<==>	$0 \le e = d = r \le h-1$
PA-->EA-->RD	<==>	$0 \le e < d = r \le h-1$
PA-->ED-->RD	<==>	$0 \le e = d < r \le h-1$
PA-->PD-->RD	<==>	$0 \le d < e = r \le h-1$
PA-->PD-->ED-->RD	<==>	$0 \le d < e < r \le h-1$

Patterns of line evolution

1. When the enhancement time e, occurs before the digitization time, d the network, is said to follow the pattern A, corresponding to a sequence of states

PA-->EA-->ED-->RD

or

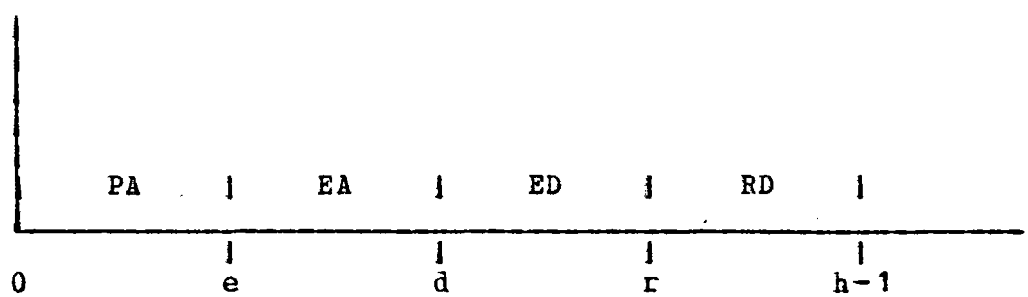

Fig .3 Line evolution as Pattern A.

2. When digitization time d, occurs before the enhancement time e, the network evolution policy belongs to a class which is distinct from A. This new way is referred to as pattern B and corresponds to a sequence of states as

PA-->PD-->ED-->RD

or

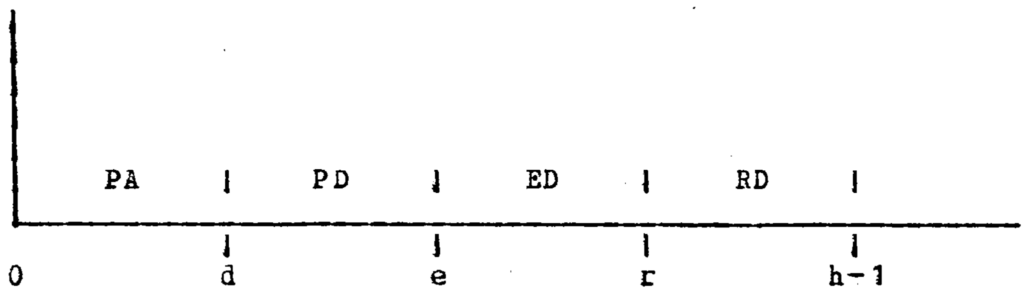

Fig .4 Line evolution as pattern B.

A mathematical way to differ these two patterns in the economic analysis given here is to use a function defined as

$$GF(x) = \begin{pmatrix} 1 \text{ if } x>0 \\ 0 \text{ if } x=0 \\ 0 \text{ if } x<0 \end{pmatrix}$$

And then $GF(d-e)>0$ implies that pattern A is considered but if $GF(e-d)>0$ that is to mean pattern B is under way. Another use of this function is to relax inequalities. As an example assume a variable takes value x when $r<h$ and is zero otherwise. This is equivalent to the variable equal $x*GF(h-r)$.

Two useful parameters are defined now.

The minimum value of e,d

$$m=\min(e,d)$$

The maximum value of e,d

$$l=\max(e,d)$$

Using these parameters the line evolution is depicted in Fig .5

$$PA \dashrightarrow \begin{pmatrix} PD \\ EA \end{pmatrix} \dashrightarrow ED \dashrightarrow RD ,$$

or

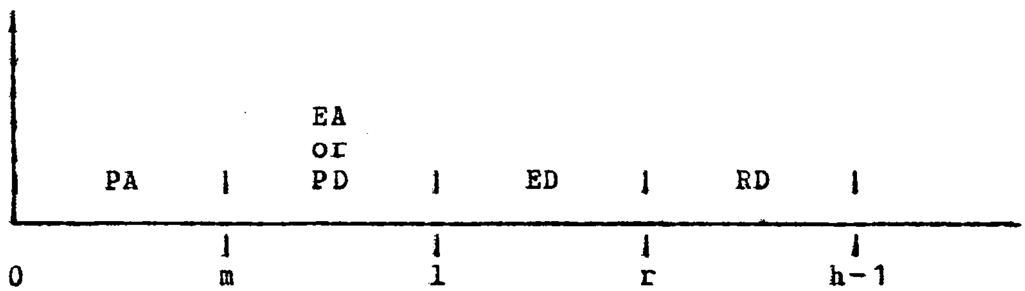

Fig .5 Line evolution

Accordingly, all strategies can be expressed in terms of the three time variables e, d, r. Also the cost function can be expressed, for a strategy, by NPV/L(d,e,r) see section 4.

SECTION-3 LINES IN EACH STATE.

3.1 LINES IN EACH EVOLUTION STATE.

POTS Analog lines (PA)

The characteristics of the network in this state are analog switching and transmission of messages. Voice channels dominate here. The terminal is the telephone set for speech communication. Signals going through the loop to Central Office are analog; and feeder, distribution cables are common copper cables. The Central Office switches are of analog type mostly mechanical. In this analysis all analog switches are of the same type as far as costs are concerned.

The number of PA(k) lines, at year k, is computed by summing up all lines digitized, enhanced, or replaced in the following years up to year h-1. The upper limit of summations are extended to h to include lines which partially or have never been modernized. That means they did not change state until the end of study period. This kind of evolution was defined as line semi-evolution.

$$PA(k)=\sum_{r=max(e,d)}^{h}\ \sum_{e=k+1}^{h}\ \sum_{d=k+1}^{h} N(e,d,r)$$

With k=0,1,...h-1

POTS Digital lines (PD)

As a first step, digital processing is introduced in the Central Office. Digital Switch, also called time switch, replaces the conventional analog switch.

The loop retains its analog status, so adapters are necessary to connect outside network to C.O. Therefore a peripheral interface is used for analog to digital conversion and signalling

In this state the number of new POTS Digital lines introduced at a year k some time inside the study period are the lines digitized this year and enhanced or replaced later till year h-1. Again upper limit is h to include possible semi-evolved lines.

So the new introduced lines are

$$New\text{-}PD(k)=\sum_{r=e}^{h}\ \sum_{e=k+1}^{h} N(e,k,r)$$

With k=0,1,...h-1

The total number of POTS Digital lines for year k must include all lines evolved to digital before year k but not yet replaced or enhanced

$$PD(k) = \sum_{r=e}^{h} \sum_{e=k+1}^{h} \sum_{d=0}^{k} N(e,d,r)$$

with k=0,1,2...h-1

Enhanced Analog lines (EA).

A step beyond conventional services was made recently, by enhancing the potentiality of the loop to meet the new demand, offering Narrow Band services and, of course, telephony.

Electronic loop techniques have been introduced. The efficiency is also improved. In this stage the network permits voice and data transmission, provided the required interfaces are introduced.

For this model, the new introduced lines at year k are obtained by summing up lines enhanced year k and digitized or replaced in the following years up to h, including semi-evolved lines. So

$$New\text{-}EA(k) = \sum_{r=d}^{h} \sum_{d=k+1}^{h} N(k,d,r)$$

Total number of enhanced analog over year k, must include lines enhanced up to year k and evolved further after year k

$$EA(k)=\sum_{r=d}^{h}\sum_{e=0}^{k}\sum_{d=k+1}^{h} N(e,d,r)$$

With k=0,1...h-1

Enhanced Digital lines (ED)

Enhancement is also accomplished by introducing the fully digital network from end to end.

In this stage the number of new ED lines for year k is constituted from lines digitized up to year k and replaced sometime in future. Summations go up to year k to include the hypothetically evoluted lines at year k.

$$New\text{-}ED(k)=\sum_{r=k+1}^{h}\sum_{d=0}^{k} N(k,d,r)$$

With k=0,1...h-1

So all ED lines over year k must include all lines enhanced before year k but not yet transformed to another state.

$$ED(k) = \sum_{r=k+1}^{h} \sum_{e=0}^{k} \sum_{d=0}^{k} N(e,d,r) \qquad k=0,1...h-1$$

Replaced Digital lines (RD).

This is the last step of the network evolution. All or a large number of elements composing the network are replaced by new systems. This system provides message telephone services, business services and new wide band services according to demand. Network is based on advanced technologies characterized by its simplicity and greater flexibility.- Sophisticated terminals may be connected through lightguides to computers to satisfy the demand.

For this stage the new introduced RD lines at year k are comprised of lines enhanced or digitized some year prior to k

$$New\text{-}RD(k) = \sum_{e=0}^{k} \sum_{d=0}^{k} N(e,d,k) \qquad k=0,1...h-1$$

So the gross number of RD over year k shoud include the lines replaced over all previous years prior to k.

$$RD(k) = \sum_{r=\max(e,d)}^{k} \sum_{e=0}^{k} \sum_{d=0}^{k} N(e,d,r) \quad k=0,1\ldots h-1$$

3.2 LINES IN EACH SERVICE CATEGORY

Services can be identified in a number of different ways. For the model considered here, service is defined in terms of the nature of the signal being transmitted, and the type of connection involved. The broad classification adopted here is the POTS, enhanced and replaced services.

POTS services

POTS or voice service is the well known telephone system with speech and very low speed data capabilities. It may have call handling features, call forwarding, or redirection, call back, priority calling, conferance calling, collect calling etc.

For the model described here the number of lines in this category at year k are given by:

$$P(k)=PA(k)+PD(k)$$

or

$$P(k)=\sum_{r=\max(e,d)}^{h}\ \sum_{e=k+1}^{h}\ \sum_{d=0}^{h} N(e,d,r)$$

where k=0,1,2,...h-1

Enhanced services.

The enhanced also called narrow band + POTS services offer voice, data, voice/data services. The data services may be subdivided into low speed and high speed data.

Some of the data services are

Data. The first group is the low speed data services (<9.6 Kbps) and include inquiry response, data entry/ collection, administrative message, computer assistant instraction.

The second group i.e. high speed data services (>9.6 Kbps) and include Remote batch distributed computing, Electronics document transmission, Graphics, Computer interface.

Voice/Data. These services include the alternate or simultaneous use of voice and data communications. Some services are Real time selective call acceptance, Home banking, Remote metering and control, Home information, Video shop at home, Adult education.

For the model assumed here the number of Enhanced lines at year k are given by

$$E(k)=EA(k)+ED(k)$$

or

$$E(k)=\sum_{r=\max(d,k+1)}^{h}\sum_{e=0}^{k}\sum_{d=0}^{h}N(e,d,r)$$

<u>Replaced services</u>.

Replaced or wideband services are primarily video services such as video phone, CATV, switched video, and Video data services such as network warehousing. Wibeband services include narrowband services.

The number of replaced lines at year k is given by:

$$R(k)=RD(k)$$

or

$$R(k)=\sum_{r=\max(e,d)}^{k}\sum_{e=0}^{k}\sum_{d=0}^{k}N(e,d,r)$$

Where $k=0,1,2...h-1$

SECTION. 4 ECONOMIC ANALYSIS

The purpose of this study is to determine the incremental effect of a proposed project on the cash flows of the firm. These cash flows are then mapped into a scalar quantity referred to as an Economic evaluator. The method employed here is the cash flow method and the considered evaluator is the Net Present Value (NPV).

Cash flows associated with the project are the basic elements of this analysis.

1. Capital costs
2. Expense costs
3. Revenues
4. Salvage

Other flows of funds associated with the project such as Income Tax, cost of Removal indirectly appear in the study. Also the Digital switch start-up costs are consolidated with the capital costs. This approximation is necessary to keep the model simple without actually affecting the validity of results.

4.1 CAPITAL COSTS

Capital costs for POTS analog (PA)

In this model it is assumed that there is no growth of demand and therefore no new connections. According to this simplification there were some analog lines in the beginning of study period that may undergo modernization. Thus capital costs for this existing analog equipment are zero

Capital costs for Pots Digital (PD)

This is a possible network state. In the analog environment a digital switch is introduced replacing the analog one. Line conditioning may also be required to meet transmission objectives.

Then present worth of capital costs are

$$(CPD\_L+CPD\_S)*w**(-d) \qquad 0 \le d < e \le r \le h$$

The digitization time d is strictly less than e to indicate that the network follows the pattern B evolution.

Then previous equation can be put in the form

$$(CPD\_L+CPD\_S)*w**(-d)*GF(e-d) \qquad 0 \le e,d \le r \le h$$

Capital cost for Enhanced analog lines (EA)

From the Pots analog state instead of going to PD an immediate service enhancement may be attractive. The Narrow Band new services are accomplished by incorporating interefaces near the terminals and Central Offices.

So present Capital costs are given by

$$(CEA\_T+CEA\_L+CEA\_S)*w**(-e) \qquad 0\leq e<d\leq r\leq h$$

This state is included when pattern A is under way. So the above form is rewritten as

$$(CEA\_T+CEA\_L+CEA\_T)*w**(-e)*GF(d-e) \qquad 0\leq e,d\leq r\leq h$$

Capital costs for enhanced digital lines (ED)

In this step of the evolution an interface for terminal and Central Office switch is required, to make the network compatible with the digital switch or the enhanced environement, actually being dependent on the previous state.

Present Capital costs for this state are.

$$(CED\_T+CED\_L+CED\_S)*w**(-1) \qquad 0\leq e,d<r\leq h$$

This can be put in the form

$$(CED\_T+CED\_L+CED\_S)*w**(-l)*GF(r-l) \quad 0\leq e,d\leq r\leq h$$

With l=max(e,d) is used because transition towards ED occurs after enhancement and digitization.

Also GF function appears to assure that the transition will include the ED state, otherwise these costs must be zero. We also expect zero costs if this transition occurs outside the study period time.

Capital costs for replaced digital lines (RD)

Towards this final state a new terminal, a Wide Band loop and new advanced interfaces in digital machine at Central Office are introduced.

Then the present worth of capital expenses are

$$(CRD\_T+CRD\_L+CRD\_S)*w**(-r) \quad 0\leq d,e\leq r<h$$

This can be put as

$$(CRD\_T+CRD\_L+CRD\_S)*w**(-r)*GF(h-r) \quad 0\leq e,d\leq r\leq h$$

It is also assumed here that the start-up costs are not a separate capital investment

Function GF is there to exclude the capital cost for case of this service to come into operation after the study period time (Semi-evolution case).

Then gross capital expenditures for an evolving line over the study period is expressed as

$$CAPITAL = CPD*w^{-d}\ GF(e-d) + CEA*w^{-e}\ GF(d-e) +$$

$$CED*w^{-l}\ GF(r-l) + CRD*w^{-r}\ GF(h-r)$$

With $0 \leq e, d \leq r \leq h$

4.2 REVENUES

To continue operating the firm must obtain certain revenues to cover costs and retain an objective profitability level. The revenues depend only upon the service offered and not on the particular network state. This dependance on the service category results to:

Revenues PA = Revenues PD = RP

Revenues EA = Revenues ED = RE

Revenues RD = = RD

Revenues are evaluated from lines available in the study period. Therefore lines in any service cetegory contributes no revenues for year h. So the semi evolved lines do not give any revenues from their final evolution occured at time h.

An economic analysis is presented next, based on the present worth of cumulated revenues throughout the study period.

Revenues from POTS services

The present worth of revenues per line obtained from POTS services are given by

$$\sum_{k=0}^{e-1} RP*w^{-k} \qquad 0 \le e, d \le r \le h \,,\; e<h$$

this yields to

$$RP*f \quad - \quad RP*f*w^{-1} \qquad 0 \le e, d \le r \le h$$

f is defined in paragraph 2.2

Revenues from Enhanced services

During the enhanced period of a line the present worth of revenues are:

$$\sum_{k=e}^{r-1} RE*w**(-k) \qquad 0 \le e<r<h \;,\; 0 \le d \le r \le h$$

this can be put in the form

$$RE*f*w^{-e} \quad - \quad RE*f*w^{-r} \qquad 0 \le e, d \le r \le h$$

I should mention here that if the replacement occurs the same year with the enhancement there are no revenues from the enhanced lines. Also these revenues are zero if the enhancement time is equal to h (semi evolved lines).

Revenues from replaced services

Revenues will also be obtained from the wideband services and their present worth per line is

$$\sum_{k=r}^{h-1} RD*w^{-k} \qquad 0\leq e,d\leq r<h$$

This is rewritten as

$$RD*f*w^{-r} - RD*f*w^{-h} \qquad 0\leq e,d\leq r\leq h$$

In the preceeding study it was always assumed that the replacement time comes not prior to digitization or enhancement time.

So the total present worth of revenue in flows is expressed as

$$REVENUES= f*(RP+(RE-RP)*W^{-e} +(RD-RE)*w^{-r} -RD*w^{-h})$$

Where $0\leq e,d\leq r\leq h$

4.3 SALVAGE FLOWS.

In this section the Net Salvage capital from removed equipment is analysed. Net salvage is defined the gross salvage less the removal costs, and it is treated as a capital cash inflow. Depending upon the network state the salvage from loops terminals, and switches is different. The saving obtained from one line when it is removed is computed below. This analysis is based on the theory presented in section 2.3 Network evolution policies.

A point which must be stressed, is that there is no salvage benefits from semi-evolved lines from their final evolution stage because that occurs outside the study period time.

Assumptions which will be made in this paragraph are based on present speculations of future systems, and may not hold for any general case. So, if the assumptions are changed new salvage dynamics may be necessary.

Salvage from terminal removals.

There are certain components, usually interface electronics, removed from the terminal section of a line every time the line changes state in its evolution, but the salvage benefit from these components is assumed to be negligible.

Significant savings from terminal removal, are obtained when the terminals are replaced with new ones at time r, the replacement time, which is the time when the network enters the final state.

Analytically the present worth of salvage from each of PA, PD, EA, ED terminal states is given below. The RD terminals is assumed that they are not salvaged.

For terminal PA is SPA_T*w**(-r) if $0 \le d, e \le r < h$

or

SPA_T*w**(-r)GF(h-r) $0 \le d, e \le r$

For terminal EA is SEA_Tw**(-r) if $0 \le e < d \le r < h$

or

SEA_Tw**(-r)GF(d-e)GF(h-r) $0 \le d, e \le r$

For terminal PD is SPD_T*w**(-r) if $0 \le d < e \le r < h$

or

SPD_T*w**(-r)GF(e-d)GF(h-r) $0 \le d, e \le r$

For terminal ED is SED_Tw**(-r) if $0 \le d, e < r < h$

or

SED_Tw**(-r)GF(r-1)GF(h-r) $0 \le d, e \le r$

<u>Salvage from loop removals</u>.

The loop in general is not a salvaged asset because even if some components have some value the removal costs are very high and therefore the net benefit low.

However, it is assumed here that some salvage benefit may be obtained from the ED loop, due to new electronics which it includes, when it is removed at the replacement time r. So the present worth of this cash inflow is

$$SED\_Tw^{**}(-r) \quad \text{if} \quad 0 \le d, e < r < h$$

or

$$SED\_Tw^{**}(-r)\,GF(r-1)\,GF(h-r) \qquad 0 \le d, e \le r$$

<u>Salvage from switch removals</u>.

A potential contribution to the firm's income is obtained when an analog switch is removed. Depending on the network state this salvage differs. For the PA, EA states where the analog switch is removed is considerably high and this occurs at time d, the digitization time. So the present woth of cash inflows from these switches is

For switch PA is $SPA\_Sw^{**}(-d)$ if $0 \le d, e \le r \le h$, $d < h$

or

$$SPA\_Sw^{**}(-d)\,GF(h-d) \qquad 0 \le d, e \le r$$

For switch EA is $SEA\_Sw^{**}(-d)$ if $0 \le e < d \le r \le h$, $d < h$

or

$$SEA\_Sw^{**}(-d)\,GF(d-e)\,GF(h-d) \qquad 0 \le d, e \le r$$

For the PD, ED states some salvage is obtained from the electronics in the central office at time r. The present worth is given from

For electronics PD is $SPD\_Sw^{**}(-r)$ if $0 \le d < e \le r < h$

or

$$SPD\_Sw^{**}(-d)\,GF(e-d)\,GF(h-r) \qquad 0 \le d, e \le r$$

For electronics ED is $SED\_Sw^{**}(-r)$ if $0 \le d, e < r < h$

or

$$SED\_Sw^{**}(-r)\,GF(r-l)\,GF(h-r) \qquad 0 \le d, e \le r$$

So the total net salvage is given by

$$\begin{aligned} SALVAGE = {} & (SPA\_T+SEA\_TGF(d-e) + (SPD\_T+SPD\_S)\,GF(e-d) + \\ & (SED\_S+SED\_L+SED\_T)\,GF(r-l)\,)\,GF(h-r)\,w^{**}(-r) + \\ & (\,(SPA\_S+SEA\_SGF(d-e)\,)\,GF(h-d)\,w^{**}(-d)\,. \end{aligned}$$

Given $0 \le e, d \le r \le h$

and $l = \max(e, d)$

4.4 EXPENSE COSTS

Expense costs for (PA) lines

The expense cost for a line in PA state is the cumulated sum of the expenses for terminal, loop, and switch throughout the life of the POTS analog network normalized to time zero. So the present value is

$$\sum_{k=0}^{m-1} (EPA\_T+EPA\_L+EPA\_S)\cdot w^{-k} \qquad 0<e,d\leq r\leq h$$

Where m is the time the network moves out of the PA state see Fig .5

Then this can be put as

$$EPA*f - EPA\cdot f\cdot w^{-m} \qquad 0\leq e,d\leq r\leq h$$

Where EPA=EPA_T+EPA_L+EPA_S

And m=min(e,d).

Expense costs for (PD) lines.

The expense costs for POTS digital are included in if the line evolution follows pattern B and it is to assure that line reaches this state.

So the present value of expense costs is

$$\sum_{k=m}^{l-1} (EPD\_T+EPD\_L+EPD\_S)\, w^{-k} \qquad 0\leq d<e\leq r\leq h$$

Time m network enters state PD,
time l departs from state PD (Fig. 5)
Then

$$EPD\cdot f\cdot GF(e-d)\cdot w^{-m} \;-\; EPD\cdot f\cdot GF(e-d)\cdot w^{-l}$$

With $0\leq e,d\leq r\leq h$
And EPD=EPD_T+EPD_L+EPD_S

Expense costs for (EA) lines.

The expense costs for the enhanced analog lines are included if the line evolution follows the pattern A. More specifically the evolution policy is assumed to be PA-->EA-->ED-->RD. Then the present worth of the expense cost outflows is

$$\sum_{k=m}^{l-1} (EEA\_T+EEA\_L+EEA\_S) w^{-k} \qquad 0 \le e < d \le r \le h$$

Time m network enters state EA

Time l departs from state EA (Fig. 5)

Then

$$EEA \cdot f \cdot GF(d-e) \cdot w^{-m} \quad - \quad EEA \cdot f \cdot GF(d-e) \cdot w^{-l}$$

With $0 \le e, d \le r \le h$

And $EEA = EEA\_T+EEA\_L+EEA\_S$

Expense costs for ED

The expense costs for enhanced digital lines starts at time l=max(e,d), the time the network is expanded to include the narrow band services untill the time r when is replaced.

So the present value of expense costs is

$$\sum_{k=l}^{r-1} (EED\_T+EED\_L+EED\_S) w^{-k} \qquad 0 \le e, d \le r < h$$

See fig .5. Then the above summation can be put as

$$EED\cdot f\cdot GF(h-r)\cdot w^{-1} \quad - \quad EED\cdot f\cdot GF(h-r)\cdot w^{-r}$$

With $0\le e,d\le r\le h$

And $EED=EED\_T+EED\_L+EED\_S$

Expense costs for RD

The expense costs for replaced digital lines starts at time r, the replacemnt time, and goes up to the year h-1 which is the end of study period.

So the present value of expense costs is

$$\sum_{k=r}^{h-1} (ERD\_T+ERD\_L+ERD\_S)\cdot w^{-k} \qquad 0\le d<e\le r\le h$$

or this can be put as

$$ERD\cdot f.\, w^{-r} \quad - \quad ERD.f.w^{-h}$$

With $0\le e,d\le r\le h$

And $ERD=ERD\_T+ERD\_L+ERD\_S$

So the total expense costs are given by

$$\text{EXPENSES}=EPA*f+(-EPA*f+EPD*fGF(e-d)+EEA*fGF(d-e))*w^{-m}$$

$$+(-EPD*fGF(e-d)-EEA*fGF(d-e)+EED*f)*w^{-l}$$

$$+(ERD-EED)*f*w^{-r}-ERD*f*w^{-h}$$

With $f=1/(1-w**(-1))$

And $0\leq e,d\leq r\leq h$

4.5 COST FUNCTION

The cost function is the Net Present Value per Line NPV/L(e,d,r) and is given by

$$NPV/L(e,d,r) = -CAPITAL + REVENUES + SALVAGE - EXPENSES$$

Where

$$CAPITAL = CPD*w^{-d}\ GF(e-d) + CEA*w^{-e}\ GF(d-e) + CED*w^{-1}\ GF(r-1) + CRD*w^{-r}\ GF(h-r)$$

$$REVENUES = f*(RP + (RE-RP)*W^{-e} + (RD-RE)*w^{-r} - RD*w^{-h})$$

$$SALVAGE = (SPA\_T + SEA\_TGF(d-e) + (SPD\_T + SPD\_S)\ GF(e-d) + (SED\_S + SED\_L + SED\_T)\ GF(r-1))\ GF(h-r)\ w**(-r) + ((SPA\_S + SEA\_SGF(d-e))\ GF(h-d)\ w**(-d).$$

$$EXPENSES = EPA*f + (-EPA*f + EPD*fGF(e-d) + EEA*fGF(d-e))*w^{-m} + (-EPD*fGF(e-d) - EEA*fGF(d-e) + EED*f)*w^{-1}$$

$$+(ERD-EED)*f*w^{-r} - ERD*f*w^{-h}$$

With $0 \leq e, d \leq r \leq h$

The Net present Value (NPV) indicator is a generalization of other indicators. For example the Present Worth of Annual Costs (PWAC) can be obtained by setting the revenue terms to zero ang taking the negative of the resulting NPV.

Also the IFC can be obtained as well as AC by appropriate selection of the discount factor.

SECTION -5 MATHEMATICAL ANALYSIS.

5.1 GLOBAL COST FUNCTION AND PHYSICAL CONSTRAINTS

A general expression for the net present value per line function NPV/L(e,d,r) in terms of line evolution variables has been derived in the previous section. Accordingly the NPV for all such lines with the assumed evolution e,d,r is given by

$$N(e,d,r)*NPV/L(e,d,r)$$

Where N(e,d,r) indicates number of lines in this specific evolution

The summation over the set of all feasible strategies yields to the global NPV. So the optimal policy, is determined by the linear system of equations, among all feasible policies, which gives the largest objective function.

Function to be maximized.

$$\max_{N(e,d,r)} NPV = \sum_{r=\max(e,d)}^{h} \sum_{e=0}^{h} \sum_{d=0}^{h} N(e,d,r)*NPV/L(e,d,r)$$

Subject to constraints.

1. As was analyzed in paragraph 2.4 the line evolution variables must obey to

$$0 \leq e,d \leq r \leq h$$

2. For every e,d,r the aggregate strategic variables N(e,d,r) have to be non negative integer numbers.

$$N(e,d,r) \geq 0$$

3. Capital budget constraints are imposed on the new introduced lines, in any state throughout the study period time. A mathematical expression for that is

$$CPD*New\_PD(k)+CEA*New\_EA(k)+CED*New\_ED(k)+CRD*New\_RD(k) \leq T(k)$$

With k=0,1,2,...h-1

And New_PD(K)..etc.. are defined in paragraph 3.1 .

The coefficients CPD, CEA, CED, CRD, used in the above equation are actually the capital costs per line necessary to cover the expenses of installement and putting every line in service.

An analytical expression for that is as follows

$$CPD\sum_{r=e}^{h}\sum_{e=k+1}^{h} N(e,k,r) + CEA\sum_{d=0}^{h}\sum_{d=k+1}^{h} N(k,d,r) +$$

$$CED\sum_{r=k+1}^{h}\sum_{d=0}^{k} N(k,d,r) + CBD\sum_{d=0}^{k}\sum_{e=0}^{k} N(e,d,K) \leq T(k)$$

With k=0,1,2...h-1

4. For every year the total number of lines in each state category, or equivalently in each service category, must be equal to the initial number of lines S0.

Therefore-summing up the lines in each service category (paragraph 3.2) it yields

$$P(k)+E(k)+R(k)=S0 \quad \text{for } 0\leq k\leq h$$

Analytically this constraint is expressed as

$$\sum_{r=\max(d,e)}^{h} \sum_{e=k+1}^{h} \sum_{d=0}^{h} N(e,d,r) + \sum_{r=\max(k+1,d)}^{h} \sum_{e=0}^{k} \sum_{d=0}^{h} N(e,d,r)$$

$$+ \sum_{r=\max(d,e)}^{k} \sum_{e=0}^{k} \sum_{d=0}^{k} N(e,d,r) = S0$$

With k=0,1,2...h

This is a tedious equation but it can be simplified considerably. To obtain this two observations are critical. First, any line cannot follow simultaneously two different evolution policies, second, for every k all possible policies have to be summed up to give the total number of lines. Then the equation is independent of k and is formulated as

$$\sum_{r=\max(e,d)}^{h} \sum_{e=o}^{h} \sum_{d=0}^{h} N(e,d,r) = S0$$

For years 0,1,2,...h

5.2 METHOD OF SOLUTION.

Except for some trivial cases, it is unlikely that an analytical solution can be obtained from the problem derived previously. Special cases occur when it is assumed that some a priory information is available. This information could indicate that there is no enhancement stage, expressed as e=r, or no digitization stage, if d=r. For these cases a much simpler set of equations is obtained.

As an example assume e=r, then the formula giving the net present value per line (NPV/L) is simpified to

$$NPV/L(d,r)=a+(b*GF(r-d)+c*GF(h-d))*w^{-d}+(d*GF(h-r)+e)*w^{-r}$$

Subject to-constraints.

1. $0 \leq d \leq r \leq h$

2. $N(d,r) \geq 0$

3 $$CPD\sum_{r=k+1}^{h} N(k,r) + CRD\sum_{d=0}^{k} N(d,k) \leq T(k)$$

4. $$\sum_{r=d}^{h} \sum_{d=0}^{h} N(d,r) = SO$$

Where k=0,1,2...h.

A similar expression can be derived for d=r.

The above results indicate even for this trivial cases a mathematical approach to the problem is cumbersome, due to the dicsontinuous function GF(.).

For arbitrary time ordering of e and d the equations are difficult to analyse because of discontinuities in the objective function. An attractive approach to the solution of this problem is to use a parameter optimization technique. Moreover, since all e,d,r are constrained to lie on a limited range, and hence the number of N(e,d,r) variables is limited, a direct search maximization scheme is more suitable of this problem.

5.3 NUMERICAL RESULTS.

The numerical solution presented here is based on Linear Programming procedures of MPSX/370 (see Appendix 3) employing the revised simplex method with product form of the inverse and with bounded variables. The simplex method is based upon the fact that if there are m constraints (or rows) in the constraint matrix and these are linearly independent, then there is a set of m columns (variables) which are also linearly independent. Hence, any right hand side can be expressed in terms of these m columns (called basis). The simplex method uses these basic solutions, stepping from one to another, until a solution (called basic feasible solution) is obtained that meets all of the criteria, including the requirement that all the column values be non negative.

After the basic feasible solution is found, the simplex method steps along, examining a series of basic feasible solutions, to find one that satisfies the requirement that the value of the objective row be maximum and this is an optimal solution. There are cases where a solution does not exist, then the problem is infeasible or unbounded.

A simple iteration consists of choosing a non basic variable and having it moved in the direction that optimizes the objective function with gradient the "reduced cost" of the variable.

Results

In Fig .6 the horizontal triangular plane specifies the feasible domain of the function NPV/L(e,d,r) according to constraint $0 \leq e, d \leq r \leq h$. Computer results indicate that NPV/L(e,d,r) is concave in all triangular planes and the first nine maxima included in table .1 belong to the upper most triangular plane and the projection of their locus is the discrete points 009, 119, 229,.. 889 all of them located on the line segment starting at the vertex 009 and intersecting the opposite side.

e d r	NPV/L(e,d,r)
0 0 9	471.5
1 1 9	424.0
2 2 9	384.5
3 3 9	349.3
4 4 9	318.0
5 5 9	292.1
6 6 9	269.9
7 7 9	248.9
8 8 9	231.4

Table .1 Policy vs NPV. First nine maxima

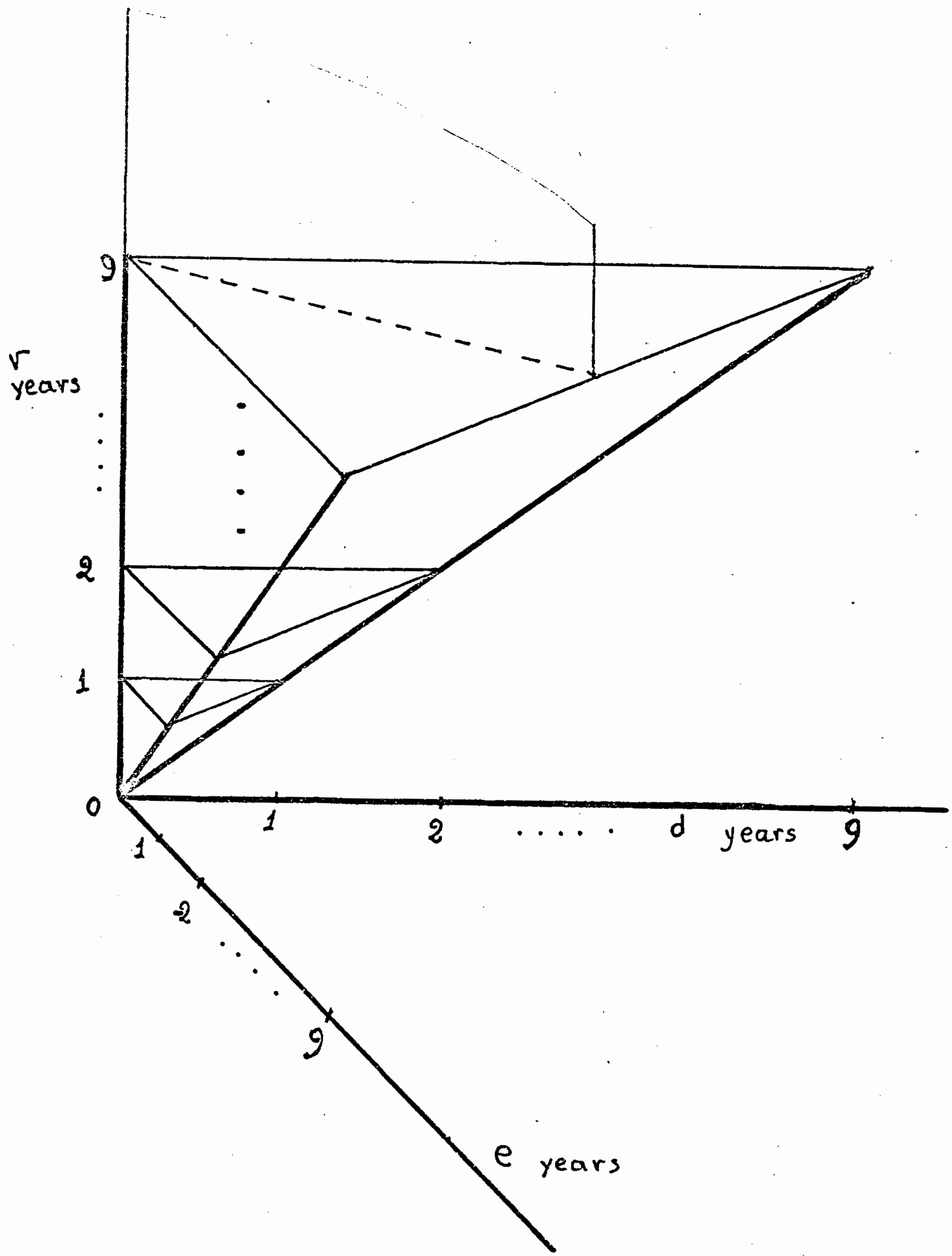

Fig .6 Feasible and optimal policies.

For the set of input parameters of Appendix 3 the linear program shows that the optimum evolution policy is

PA-->ED

and this is the case for any non zero budget per year. The number of lines evolved towards ED every year depends on the capital budget. If, say, capital was infinite then the optimum modernization policy would be d=e=0, r=h or simply immediate transition to ED state. If budget of the first year was not enough to allow all lines to reach ED state then the budget of the next year would be allocated for this, according to evolution e=d=1, r=9 the second optimum choice after e=d=0, r=9; and so on. These observations are based on results shown in table .1, where the nine first maxima of NPV/L(e,d,r) are included, with the corresponding evolution.

Table 2 indicates the number of enhanced digital lines ED(k) as a function of time and the capital budget/year as a parameter for the optimal moderization policy.

ED LINES

time \ Budget in $/year	0.00	10^4	10^5	$2.5*10^5$	10^6	$3*10^7$
0	0	200	2000	5000	20,000	50,000
1	0	400	4000	10,000	40,000	50,000
2	0	600	6000	15000	50,000	50,000
3	0	800	4000	20,000	50,000	50,000
4	0	1000	2000	25000	50,000	50,000
5	0	1200	12,000	30,000	50,000	50,000
6	0	1400	14,000	35,000	50,000	50,000
7	0	1600	16,000	40,000	50,000	50,000
8	0	1800	18,000	45,000	50,000	50,000

Table . 2 Enhanced digital lines vs

Budget constraint. Optimal policy

These results are obtained with the input parameters defined in Appendix 2. The results can be explained considering the fact that the evolution policy is PA--->ED and the Capital costs per year per line were assumed to $50. So , say, for year 0, the budget is $100,000 then there are 100,000/50= 2,000 enhanced digital introduced in every year and this is what appears on table by running the program.

Fig. 7 below gives the number of ED lines as a function of time with the capital budget/year as the varying parameter. The total number of lines is assumed to be 50,000.

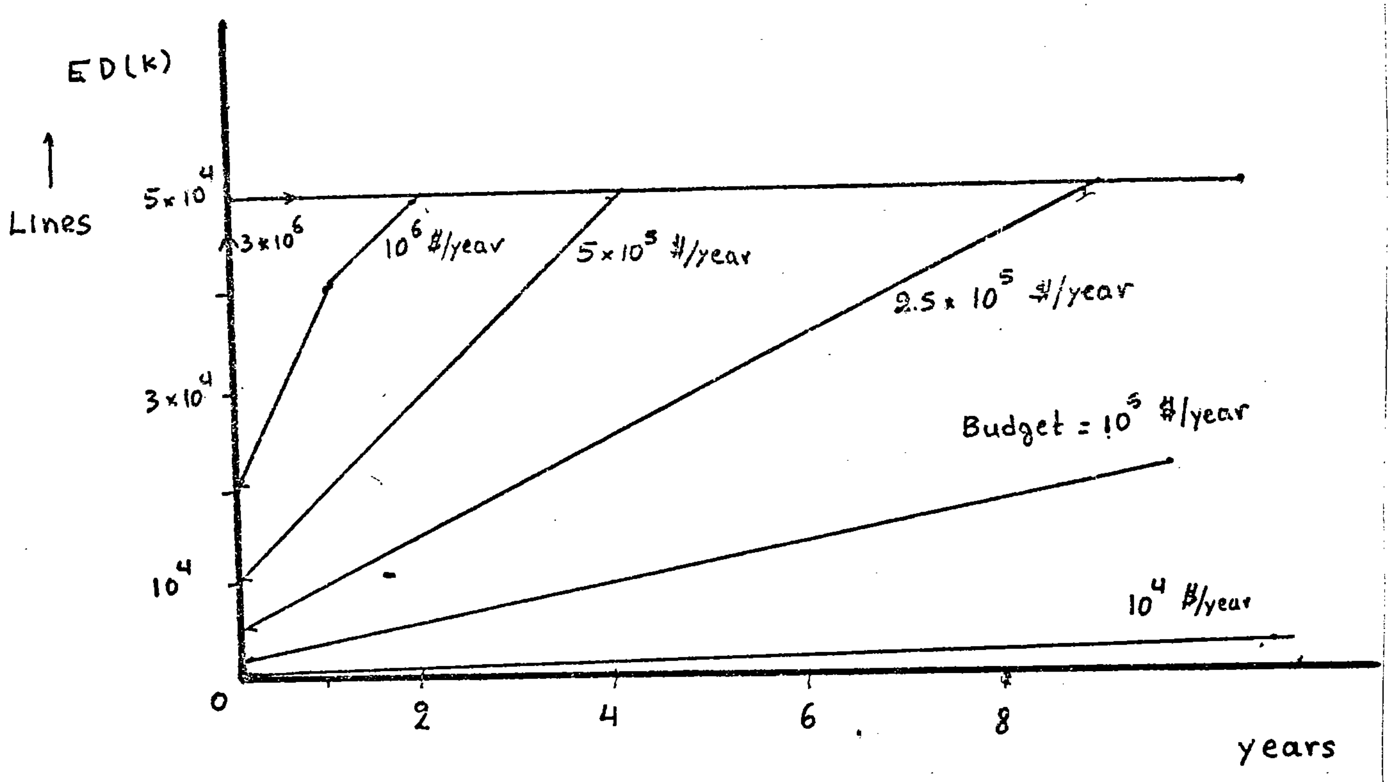

Fig .7 Optimal state evolution vs capital budget constraint.

Lines in each state. Optimum strategy.

The Job step.4 (Appendix 3) program package calculates the number of lines in each state for every year of the study period for the optimum strategy.The results shown in table 3 have been obtained for capital budget per year equal to $100,000.

Line state / time	PA	PD	EA	ED	RD
0	48,000	0	0	2,000	0
1	46,000	0	0	4 000	0
2	44,000	0	0	6000	0
3	42,000	0	0	8000	0
4	40,000	0	0	10,000	0
5	38,000	0	0	12,000	0
6	36,000	0	0	14,000	0
7	34,000	0	0	16,000	0
8	32,000	0	0	18,000	0

Table .3 Lines in each state. Optimal policy.

NPV as a function of budget per year.

The NPV depends on the capital budget allocated to new introduced lines and tends to increase as the budget increases.

Fig .8 depicts the NPV as a function of Capital budget per year.

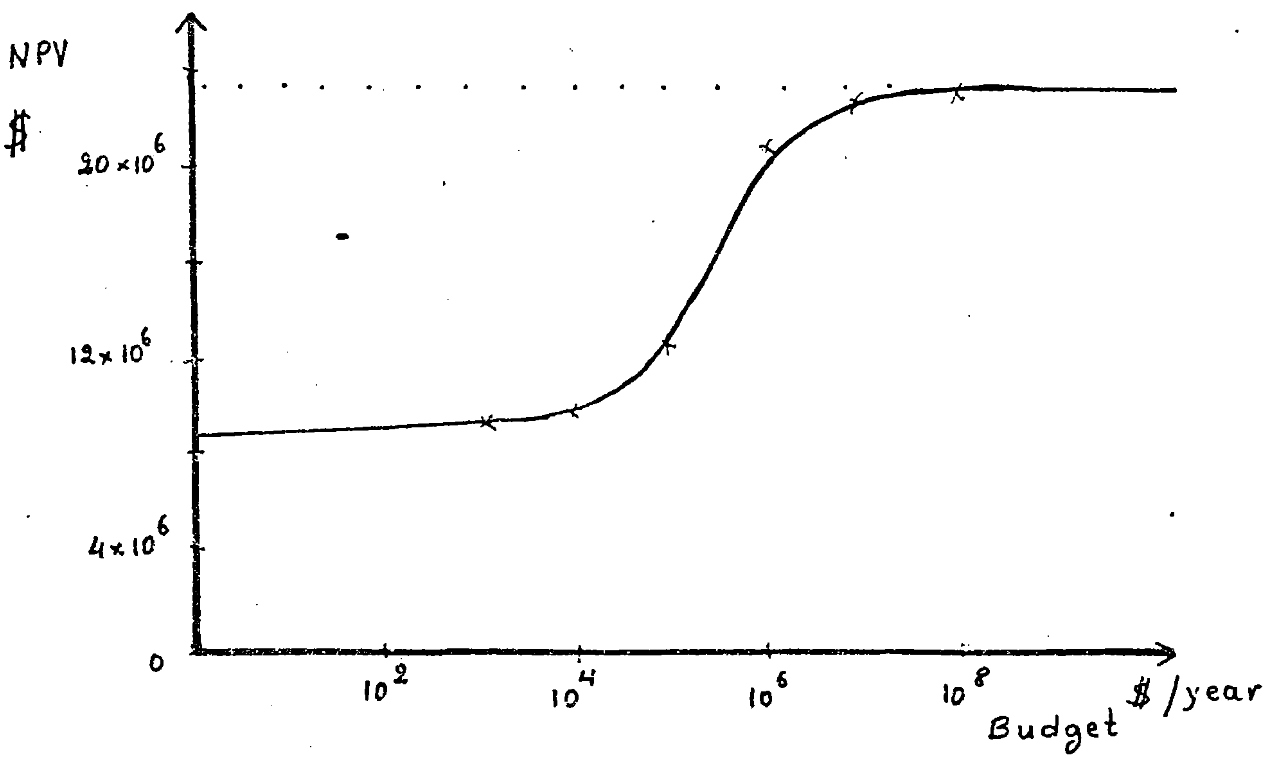

Fig .8 NPV vs budget constraint.

The NPV increases as the budget increases but for values of Budget greater than $3,000,000 the NPV retains a maximum value and thereafter remains constant. This is due to the fact that the budget is enough to enhance all lines simultaneously and therefore NPV is at its maximum value. and a further increase of the budget is of no interest.

CONCLUSION

An aggregate cost model for integrated Wire-Center modernization has been described, where this cost is the Net Present Value (NPV) and is a function of the time of digitization, enhancement, and replacement.

In this model has been assumed that the study period is discrete and finite; no overlay option; two kinds of digital switches, analog and digital; and salvage benefits are of considerable importance.

It has been demostrated that the optimal strategy for the network evolution strongly depends on the values of the input cost parameters, but trials with different data sets indicate that pure modernization policies are optimum when there are no capital budget constraints. If capital budget constraints are imposed on the lines introduced every year the best strategy is to allocate all this per year capital to the inroduction of the equipment necessary to allow the network to reach the state determined by the pure policy.

This model considers five pure policies depending on the equipment introduced or the offered service and defined as

a. PA-->PA Network retains its analog status till the end of study period.

b. PA-->EA All new introduced lines are of enhanced analog (EA) type.

c. PA-->PD Network remains analog but now part or all of it, depending on capital budget, operates on digital basis.

d. PA-->ED Part or all network, depending on capital budget, becomes enhanced digital.

e. PA-->RD Part or all network, depending on capital budget, is reformed to accomodate advanced services and therefore tends to reach the replaced digital state.

Due to discreteness of the model and the discontinuities of the evaluator (cost function) theoretical analysis has not been done. Instead, a computer aided solution has been achieved, employing an exhaustive search scheme to find the maximum value. The problem has been formulated in a way as to be possible the use of linear programming to obtain the solution. This linear programing optimization package is easily available as well as cost effective.

Generalization. It appears that while the model has telephonic origins, it is applicable to a broader class of modernization problems. For example other communication networks and perhaps transportation networks as well, although the latter example appears to be somewhat more questionable.

APPENDIX 1

Parameters used in this Linear Programming Formulation (LPF) differ from those in the report. The reason is the used computer language FORTRAN IV G1 does not allow a great deal of flexibility manoeuvering parameters.

This appendix will help to establish a link between report parameters and parameters basically employed in the MAIN program and function SNPV (net present value per line) of the JOB STEP 2.

If some variable is not included in the list below that is to mean it retains its name (given, of coarse, a change from Low Case to Upper Case letters)

Peport	Program	
h-1	HH-1	Study period
e	JE	Local strategic variables
d	JD	" " "
r	JR	" " "
m	M	Function of e,d,r
l	L	" " "
N(e,d,r)	N(JE,JD,JR)	Aggregate strategic variable

The functions that compute Revenues, Capital costs, Expense costs, and Salvage appeared in the program are repeated here to make easier the parameter correspondance.

For Revenues

REVEN=VX(11)+VX(12)*w**(-je)+VX(13)*w**(-jr)-VX(14)*w**(-MH)

Where

VX(11)=COST(10)*F=RP*F

VX(12)=(COST(30)-COST(10))*f=(RE-RP)*F

VX(13)=(COST(50)-COST(20))*F=(RR-RE)*F

VX(14)=cost(50)*F=RR*F

For Capital costs

CACOST=VX(4)*GF(JE-JD)*w**(-JD)+VX(6)*GF(JD-JE)*w**(-JE)

+VX(8)*GF(JR-L)*w**(-L)+VX(10)*GF(MH-JR)*w**(-JR)

Where:

VX(4)=COST(14)+COST(15)+COST(16)

=CPD_T+CPD_L+CPD_S

VX(6)=COST(24)+COST(25)+COST(26)

=CEA_T+CEA_L+CEA_S

VX(8)=COST(34)+COST(35)+COST(36)

=CED_T+CED_L+CED_S

VX(10)=COST(44)+COST(45)+COST(46)

=CRD_T+CRD_LC+RD_S

Expense costs

Using parameters introduced in program 'Expense costs' are rewritten as

```
EXCOST=VX(15)
        +(-VX(15)+VX(16)*GF(JE-JD)+VX(17)*GF(JD-je))*w**(-M)
        +(-VX(16)*GF(JE-JD)-vx(17)*GF(JD-JE)+VX(18))*w**(-L)
        +(vx(19)-VX(18))*w**(-JR)-vx(19)*w**(-MH)
```

Where:

```
VX(15)=VX(1)*F=(COST(1)+COST(2)+COST(3))*F
      =(EPA_T+EPA_L+EPA_S)*F

VX(16)=VX(3)*F=(COST(11)+COST(12)+COST(13)
      =(EPD_T+EPD_L+PD_S)*F

VX(17)=(VX(5)*F=COST(21)+COST(22)+COST(23))*F
      =(EPA_T+EPA_L+EPA_S)*F
```

```
VX(18)=VX(7)*F=(COST(31)+COST(32)+COST(33))*F
      =(EED_T+EED_L+EED_S)*F
```

```
VX(19)=VX(9)*F=(COST(41)+COST(42)+COST(43))*F
      =(ERD_T+ERD_L+ERD_S)*F
```

<u>Salvage</u>

Using parameters introduced in the program formulation 'Salvage' is rewritten as

```
SALVAG=(COST(7)+COST(27)*GF(JD_JE)+VX(20)*GF(JE_JD)+
       +VX(21)*GF(JR-L)*GF(MH-L)*w**(-JR)+
       +COST(9)+COST(29)*GF(JD-JE)*GF(MH-JD))*w**(-JD)
```

Where:

```
VX(20)=COST(17)+COST(19)
      =SPD_T+SPD_S
```

```
VX(21)=COST(37)+COST(38)+COST(39)
      =SED_T+SED_L+SED_S
```

```
COST(7)=SPA_T
COST(9)=SPA_S
COST(29)=SEA_S
```

APPENDIX 2

In this appendix are indicated the values of the parameters used to run the optimization program in the computer. The numerical results summarized in paragraph 5.3 are based on these values of the parameters.

Study period	h-1	=8
Discount rate	i	=0.150
Number of lines per year	S0	=50,000

Capital budget available per year assumed the same throughout the study period and takes values $0 $1000 $10,000 $10,000 $100,000 $1000,000 $3,000,000 depending on the case. The cost parameters given below indicate costs per line per year.

PA cost parameters.

EPA_S = 15.00	EPA_L = 0.00	EPA_T = 0.00
CPA_S = 0.00	CPA_L = 0.00	CPA_T = 0.00
SPA_S = 100.00	SPA_L = 0.00	SPA_T = 0.00

Revenues_PA= 50.00

PD cost parameters.

EPD_S = 5.00	EPD_L = 0.00	EPD_T = 0.00
CPD_S = 250.00	CPD_L = 0.00	CPD_T = 0.00
SPD_S = 200.00	SPD_L = 0.00	SPD_T = 0.00

Revenues_PD=50.00

EA cost parameters.

EEA_S =	15.00	EEA_L =	0.00	EEA_T =	0.00
CEA_S =	100.00	CEA_L =	0.00	CEA_T =	0.00
SEA_S =	75.00	SEA_L =	0.00	SEA_T =	0.00

Revenues_EA= 80.00

ED cost parameters.

EED_S =	5.00	EED_L =	0.00	EED_T =	0.00
CED_S =	50.00	CED_L =	0.00	CED_T =	0.00
SED_S =	35.00	SED_L =	0.00	SED_T =	0.00

Revenues_ED= 80.00

RD cost parameters.

ERD_S =	5.00	ERD_L =	0.00	ERD_T =	0.00
CRD_S =	750.00	CRD_L =	0.00	CRD_T =	0.00
SRD_S =	0.00	SRD_L =	0.00	SRD_T =	0.00

Revenues_RD= 120.00

APPENDIX 3

The Linear Programming solution to this problem is done in four Job Steps.

Step 1. Two data sets are created on a disc one to store the input data for job step 3, and the other to collect the output of job step 3. (Computer language is Job Control Language JCL)

Step 2. In this step the computer reads the input data set and calculates the cost function for every feasible policy and puts the results in a special format readable by the Job step 3 compiler. (Computer Language is Fortran IV.G1)

Appendix 1 relates parameters used in the report with those used in the program of this step.

Step 3. This control program reads the data and computes the optimum solution based on the revised simplex method with product form of the inverse and with bounded variables. (Computer language IBM Mathematical Programming System Extended/370; MPSX/370)

Step 4. The optimal solution obtained in Step 3 is used to find the number of lines in each of PA, PD, EA, ED, RD sates.

The Fig .9 below shows the CPU, and I/O+U/R accounting necessary for execution of the program (2,3,4 Job steps) as a function of the study period. The CPU charges are strongly dependent on the number of iterations has to be done for objective function to converge.

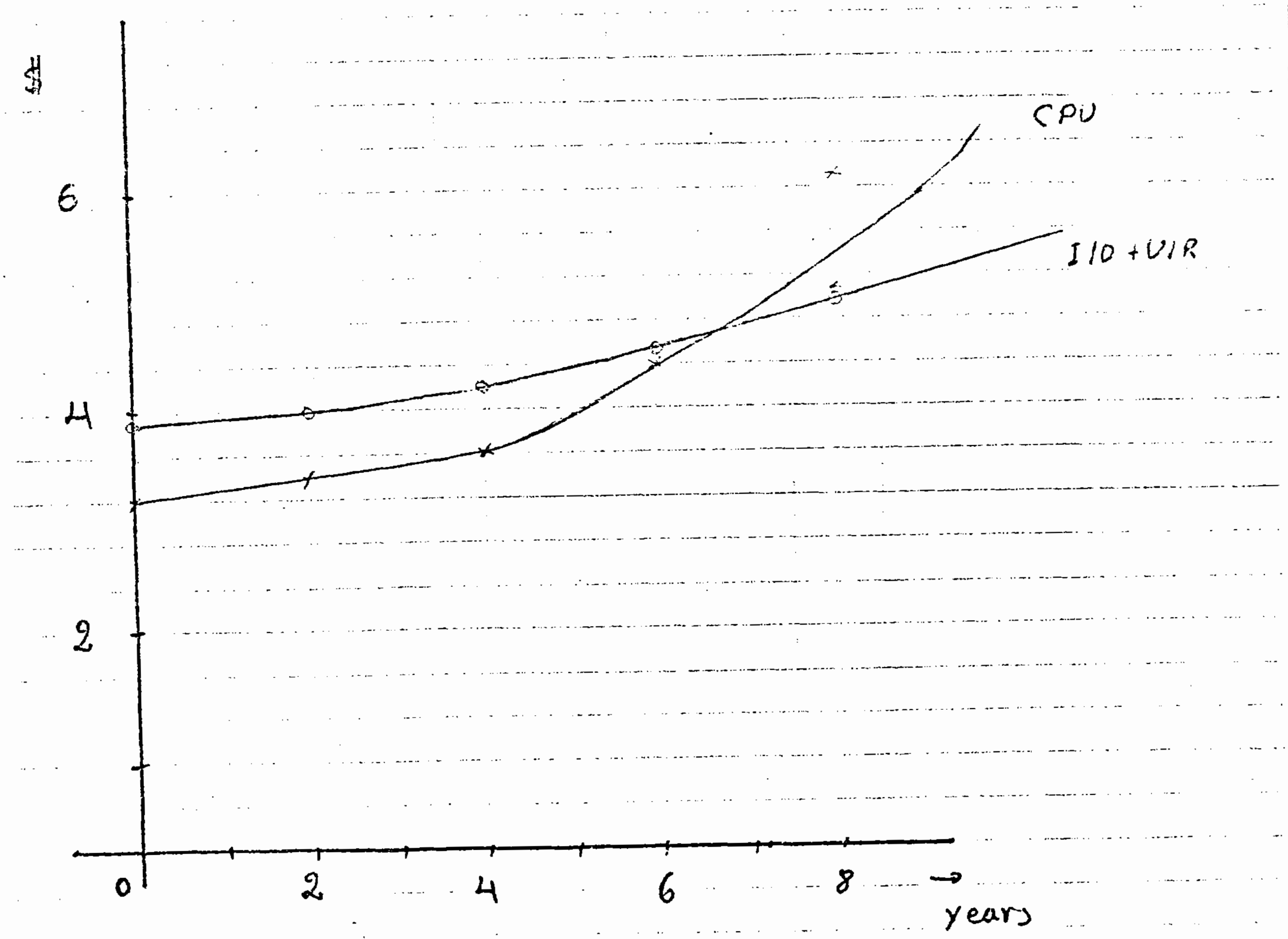

Fig .9 Computational complexity.

REFERENCES.

1. J.C.Lawson, et.al, 'The impact of ponential new telecommunication services on the structure of the local network. Proceedings of ISLS Conf. Atlanta, G.A. MARCH'78.

2. W.A.Koiser, H.T.Hagnyer, 'Digital two wire local connection providing office subscriber with speech, data and new telecommunication services'. Proceedings ISLS Conf, Atlanta, MARCH'78.

3. To_Tsuda, R.Yatsuboshi, K.Yamuchi, 'An approach to multi service subscriber loop system using packetized Voice/Data services' Proceedings ISLS Conf, Atlanta, G.A., MARCH'78.

4. L.G.Mason, J-P.Combot, 'Optimal modernization policies for telecommunications facilities', IEEE Trans on Communications, Vol.COM.27, No.3, MARCH'80.

5. J.P.Combot, L.G.Mason, 'Optimal Switch concentrator location for a telephone network' IEEE Trans on Communications, Vol.COM-27, pp36-44, JAN'79.

6. L.G.Mason, T.Gavin, 'An application of optimal control theory to a modernization problem in a telephone network' presented at the optimization days Conference, Montreal Canada, MAY'78.

7. F.Homayoun, 'Loop evolution, its dynamics and driving forces', IEEE Trans on Communications, Vol-COM.28, No.7, JULY'80.